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SMALL SCALE MIXING IN THE OCEAN*

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ABSTRACT

Observations of the temperature, salinity, and density microstructure in the ocean and fresh water lakes are discussed and compared briefly with previous explanations. None of the present theories, based on the salt fingering process, large amplitude internal waves, and shear instability, is completely satisfactory. A new laboratory experiment which exhibits regularly spaced sharp density gradients is discussed in the light of its possible relevance to the layering observed in nature. The theoretical explanation of the layering observed in the laboratory is apparently the diffusive instability (viscous overturning) proposed by McIntyre. The instability is produced by counter-gradients of angular momentum and density. A Richardson number-Prandtl number criterion is presented. For a given density gradient and ratio of viscosity to diffusivity, an increasing velocity gradient will lead to the layering mode. This suggests an observational test, perhaps most feasibly carried out in the Indian Ocean, which would study the microstructure as a function of surface wind strength and direction.

INTRODUCTION

TURBULENT mixing in the sea is one of the most important aspects of internal motions. Much of this mixing takes place on a small scale, yet we are still far from a complete understanding of the relevant processes. I would like to confine myself today to the aspect of turbulent mixing which results in the small scale structure observed in temperature, salinity and density profiles in the ocean and fresh water lakes, commonly called 'microstructure'. I shall review the observations, briefly describe some of the presently accepted explanations, and then propose a possible source of microstructure based on some recent laboratory experiments we have carried out at Harvard. Finally, I shall suggest a class of experiments for further study of microstructure which might be most feasibly carried out in the Indian Ocean.

I thank Professor Melvin Stern who first pointed out to me the relevance of the diffusive instability to the present experiment, Professor Peter H. Stone for valuable discussions, and James Duffee, and Charles Hamaker of Antioch College, and Michael Egan and Robert Weller of Harvard College for help in the experiments. This research was supported by a grant number NOOO-14-67-A-0298-0011 from the office of Naval Research to Harvard University.

SMALL SCALE MIXING IN THE OCEAN

The measurements with the new continuous reading instruments (which yield profiles of temperature and salinity to resolutions of about one metre in depth, 0.05°C in temperature, and 0.03‰ in salinity) have shown that both the temperature and the salinity have a layered structure. For example, Cooper and Stommel (1968)

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found that the main thermocline off Bermuda consists of a regular series of steps of homogeneous layers 3 to 5 metres thick in which alternate with transition layers 10 to 15 metres thick in which the temperature and salinity shift 0.03 to 0.05°C and 0.04 to 0.10°/$_{oo}$. Their horizontal extent is between 400 and 1000 metres. Pingree (1969) showed that the microstructure observed in the neighbourhood of the Mediterranean water shows phase relationships between temperature and salinity variations. The variations are such that the density profile contains less variation than either the temperature or the salinity profiles. Neal, *et al.*, (1969) (Fig. 1) found the presence of several cascaded isothermal layers in an Arctic water column. Layers between the depths of 300 and 350 metres range from two to ten metres thick, while the temperature change between adjacent layers is approximately 0.026°C. The

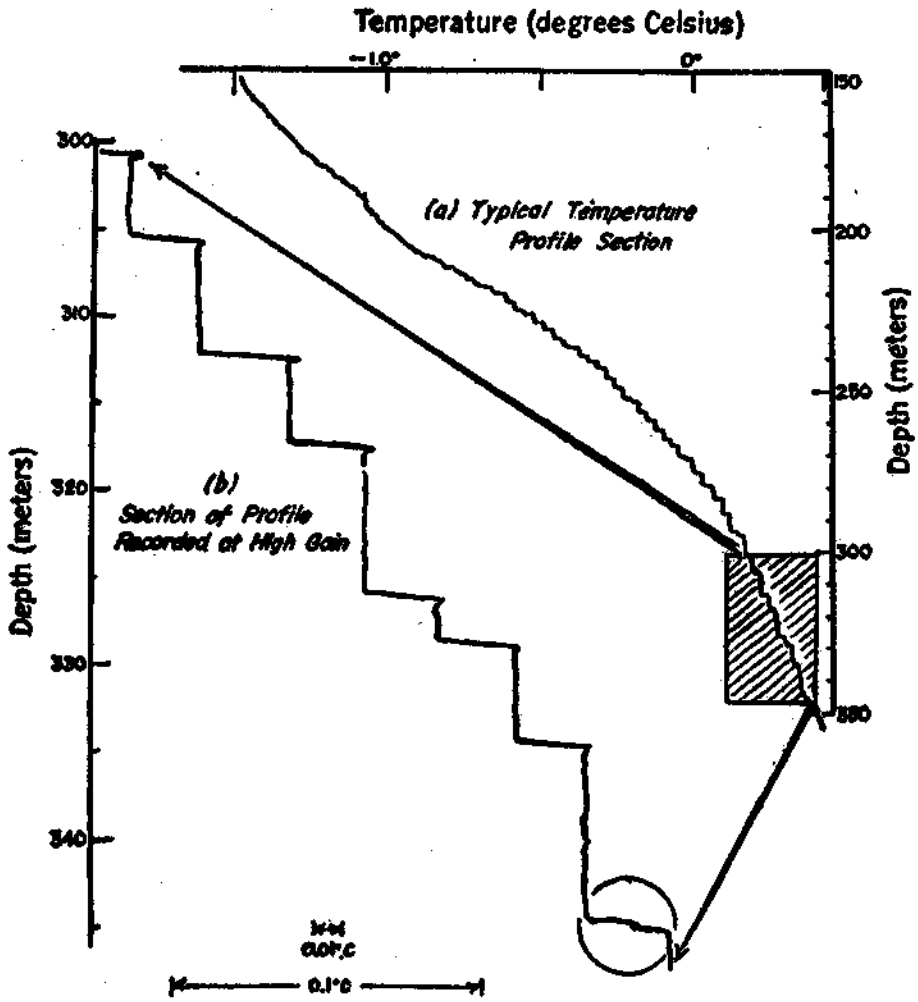

Fig. 1. Temperature profile under the Arctic Ice Island T-3. (19 March, 1969) (From Neal, *et al.*, 1969).

individual layers are isothermal to within $\pm$ 0.001°C. Tait and Howe (1968) found a series of deep layers in the Northeast Atlantic roughly mid-way between Cape St. Vincent and Madeira. The layers, formed at a depth of 1280-1500 metres, were characterised by discrete temperature and salinity steps of the order of 0.25°C and 0.44°/₀₀ and of thickness from 15 to 30 metres. Woods (1968) in his extensive study of the summer thermocline around Malta found that it is divided into layers a few metres thick and characterised by weak temperature gradients (about 10^{-3} °C/cm) and velocity shear (about 10^{-2} cm/s/cm) separated by sheets a few centimetres thick characterised by strong temperature gradients (upto 0.05 °C/cm) and shear (about 0.1 cm/s/cm). Cox, *et al.*, (1969), using a newly designed free fall instrument, were able to study spatial resolution down to a centimetre or less. The spectrum of vertical component of temperature gradient showed a sharp cut off at a wave number near one cycle/cm (Fig. 2). Small scale features were not recognizably similar over a horizontal distance of 200 metres. Some of the sharpest gradients were found in interfaces as thin as a few centimetres.

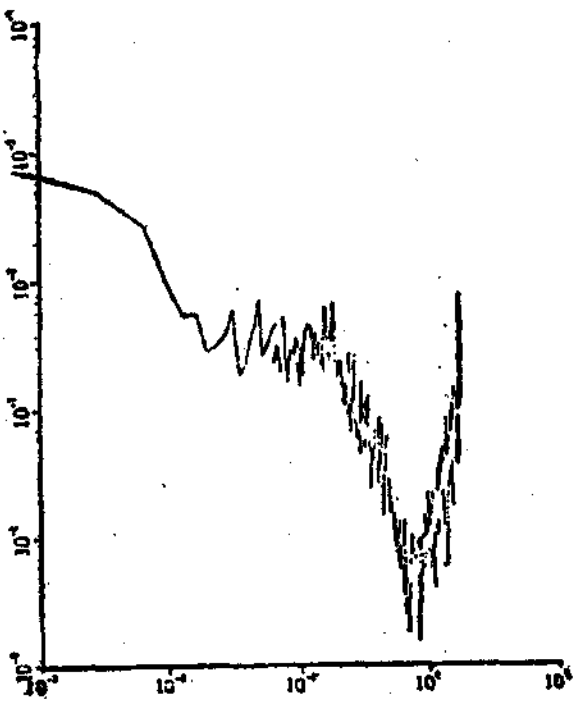

Fig. 2. The spectrum of vertical component of temperature gradient, as measured by Cox, *et al.* (1969). Vertical scale is spectral intensity in (°C/cm)² per cycle/cm. Horizontal scale is wave number in cycles/cm.

Simpson and Woods (1970) found temperature microstructure (Fig. 3) in a fresh water thermocline (Loch Ness). They state that the spectral characteristics of the vertical scale of the microstructure are essentially the same as those found by Cox, *et al.* in the Pacific and by Woods near Malta. In addition, they point out that the existence of such a microstructure in the absence of any dissolved or suspended matter suggests that salt does not necessarily play an essential part in the formation of microstructure in the oceanic thermocline.

Several explanations of the step-like structure have been put forward. The salt fingering process (Turner and Stommel, 1964 ; Turner, 1967) has produced layers in the laboratory and could exist in certain regions of the ocean, in the absence of large shears. However, this process cannot explain the microstructure observed in fresh water lakes. Orlanski and Bryan (1969) have suggested that the step structure is formed by large amplitude internal gravity waves and find that numerical experi-

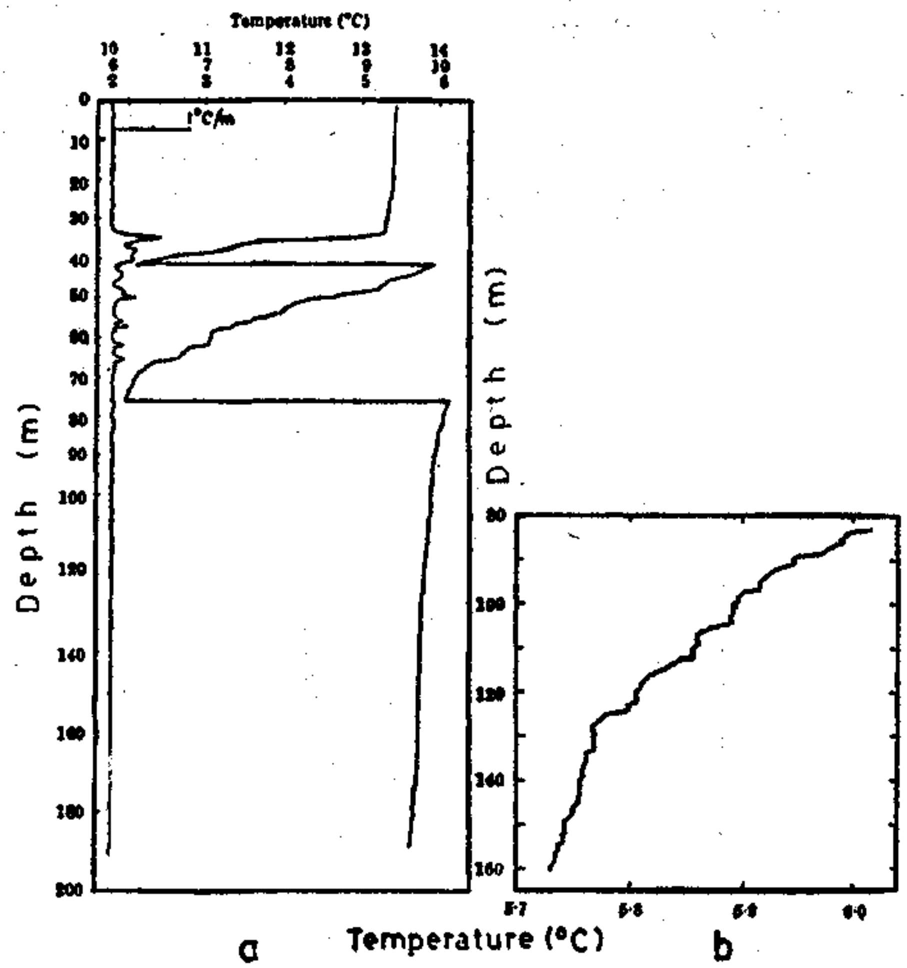

Fig. 3-a. Temperature profile (on the right) and temperature gradient profile (on the left) at Loch Ness Station 9 on September 21, 1968. b. An expanded scale plot of the temperature in the bottom layer showing a number of almost isothermal layers, differing in temperature by only a few hundredths of a degree Celsius. (From Simpson and Woods, 1970).

ments support this suggestion. They show that the measured velocity spectra in the North Atlantic contains more than enough energy in the internal wave frequency range for this type of instability to occur. Phillips (1966) has suggested shear instability. The measurements of Woods show that local shear instability can indeed give rise to Kelvin-Helmholtz billows with diameters of 20-40 cm and overturning speeds of about 0.1 cm/s.

I would like to propose here a fourth possible source of microstructure, based on some recent laboratory experiments we have carried out at Harvard. I shall describe the experiment first, then the proposed explanation, and finally speculate as to the possible oceanic relevance.

The experiment consists of a rotating cylinder filled with a stratified fluid in which relative motion is induced by a rotating disc submerged in the fluid. Large relative motion between the disc and the cylinder produces sharp vertical gradients in the density field which appear as curved horizontal sheets ; the sheets also appear near the bottom of the container when the cylinder speed is changed markedly.

The experimental arrangement is shown in Fig. 4. The cylinder is filled with a linearly stratified NaCl solution (Oster, 1965) then brought with the disc to the the desired rotation speed slowly enough so that excessive mixing does not occur (in particular, no evidence of the present instability). The disc speed (w) relative to the cylinder speed (Ω) is then varied from zero to a maximum in discrete steps and shadow graphs (Fig. 5) of the density field are photographed at each step.

No perturbations in the density field are observed when the ratio of angular velocities w/Ω (Rossby Number) is small. As the Rossby number is increased, however, sharp vertical gradients in the density field begin to appear as curved horizontal sheets above and below the disc. At first single sheets appear above and below the disc ; for higher Rossby number more sheets appear, further from the disc. A shadow graph of the density field at Rossby number of -1 is shown in Plate I. The axial symmetry of the pattern is evident. The curvature of the sheets reflects the geostrophic nature of the swirl flow. Away from the disc, the sharpness of the density gradient in the sheet decreases and the sheets become less curved, indicating a decreasing velocity gradient. The vertical distance between the sheets is several times the thickness of the sheets and is remarkably uniform. With both rotation speeds held constant, the pattern will maintain itself for at least several days, a time two orders of magnitude longer than the molecular diffusion time for salt in such thin sheets. Thus, the mixing process which is taking place continually sharpens the gradient against diffusion. Overturning motion between the sheets has been directly observed on the shadowgraph screen ; potassium permanganate particles dropped through the fluid show a step-like pattern to the swirl velocity. When the relative disc speed is reduced to zero, the sheets slowly disappear. Sheets have been observed for both positive and negative Rossby number and for three different ratios of disc to cylinder diameter.

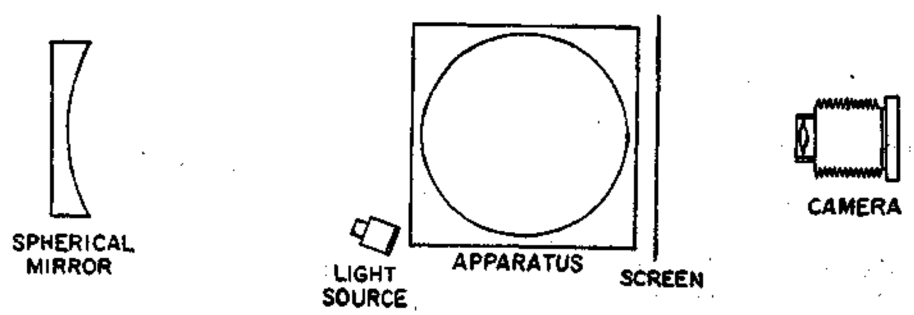

Fig. 5. The optical arrangement for observing a shadow graph of the density field. The light source is placed at the focal point of the spherical mirror.

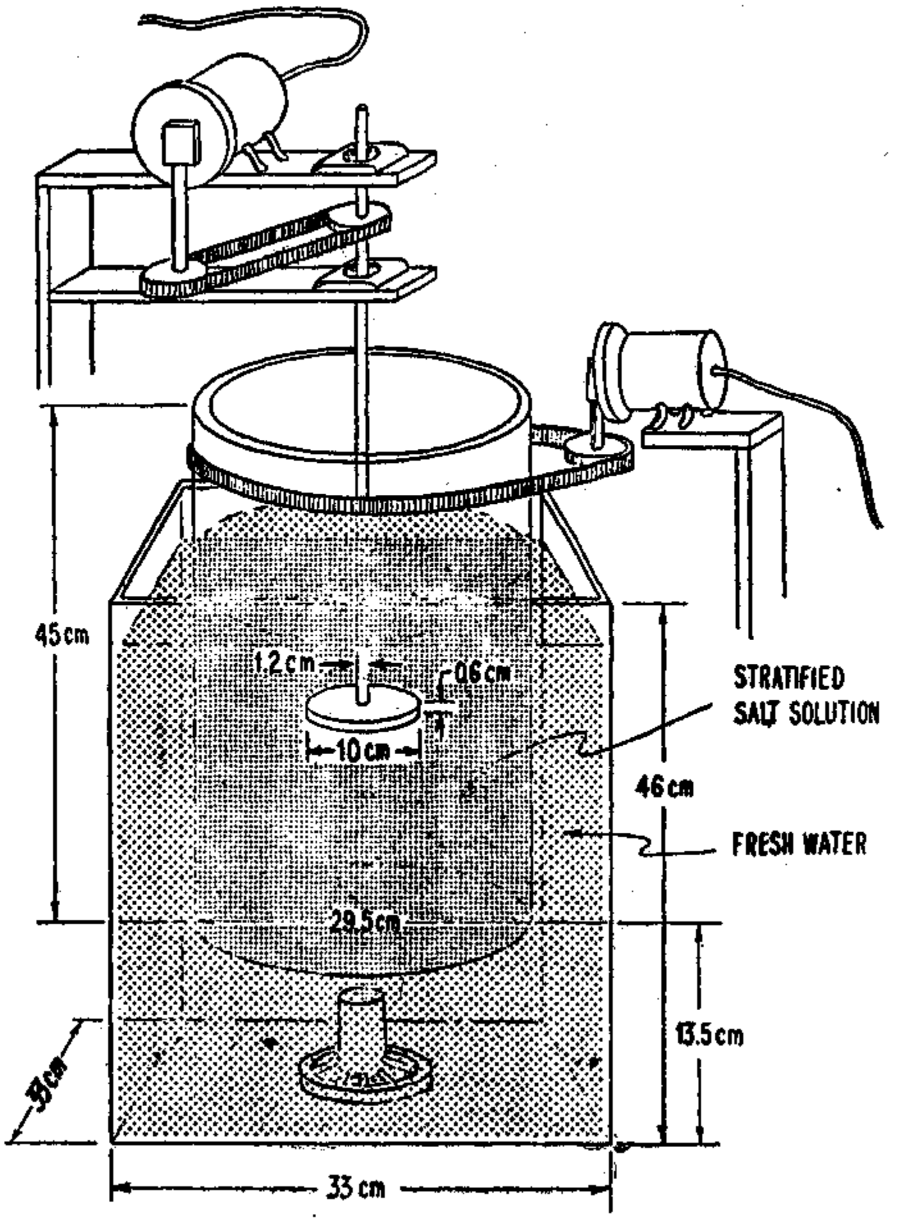

Fig. 4. Schematic diagram of apparatus. The cylinder is placed in the water-filled box in order to minimize distortion.

McIntyre (1970) has suggested that a baroclinic circular vortex can be unstable for both very large and very small Prandtl number (the ratio of the diffuse co-efficient of momentum and heat) or Schmidt number (the ratio of the diffusion coefficients of momentum and salt) σ. If the density sheets be interpreted as a finite amplitude manifestation of this instability, the present laboratory results support his conclusions for 'viscous overturning' at large Schmidt number. The physical mechanism depends on viscosity and the horizontal density gradients produced by the relative motion in the rotating system. Without viscosity, a particle of fluid displaced away from the axis of rotation (into a region of higher density) tends to return by conservation of angular momentum. When the diffusion of momentum is much larger then the diffusion of density, the viscous torque brings the particle into momentum equilibrium at its new radius; the relative absence of density diffusion causes the particle to rise vertically. Similarly, a particle moved inward will tend to sink. The overturning is the source of vorticity for the instability. The scale of the motion is determined by a balance between Coriolis and viscous forces. The instability thus produced by counter gradients of density and angular momentum is exactly analogous to that produced by counter gradients of heat and salt (Thorpe, *et al.*, 1969).

McIntyre's calculations reveal that the critical gradient Richardson number for the onset of instability for large Schmidt number is

$$\mathrm{Ri} \equiv \frac{gS}{\left(\frac{\partial v}{\partial z}\right)^2} = \frac{\sigma}{4\left(1+\frac{\partial v/\partial r}{2\Omega}\right)} \qquad (1)$$

where g is the acceleration of gravity, Ω the basic rotation rate, $\frac{\partial v}{\partial z}$ the vertical derivative of the basic velocity field, and $\partial v/\partial r$, the radial derivative of the basic velocity field. In terms of $\partial v/\partial z$, the quantity inferred from the experiment, the criterion for instability is

$$\frac{\partial v}{\partial z} > \frac{(4(1+(\partial v/\partial r)/2\Omega)\, gs)^{1/2}}{\sigma} \qquad (2)$$

The wave length of the most unstable mode is $2\pi\ (v/2\Omega)^{1/2}\ (2\Omega/(\partial v/\partial z))^{1/4}$; the growth rate is approximately $0.1(2\Omega\ (\partial v/\partial z))^{1/2}$ for the parameters relevant to the experiment.

In order to compare the parameter dependence of the observed pattern with the above theory, the disc and cylinder diameters were set at 10 and 13.8 cm respectively and experiments were run at five different values of S and three different values of σ (by varying the temperature). The results are shown in Figs. 6 and 7, where the parameters for each observed sheet are plotted on a velocity-gradient - stratification parameter plane and a velocity-gradient Schmidt number plane. The range of velocity gradients for which no sheets were observed are shown by the dotted lines. The regions in the two planes corresponding to the theoretical instability are shaded. We observe that most of the observed sheets do in fact fall in the area of predicted instability; the magnitudes of velocity gradients for which no sheets were observed also agrees reasonably well with the predicted stable regime.

The observed distance between the sheets is plotted as a function of velocity gradient and compared with theory in Fig. 8, where the distance measured is identified with one-half the wave length of the maximally growing disturbance of the theory,

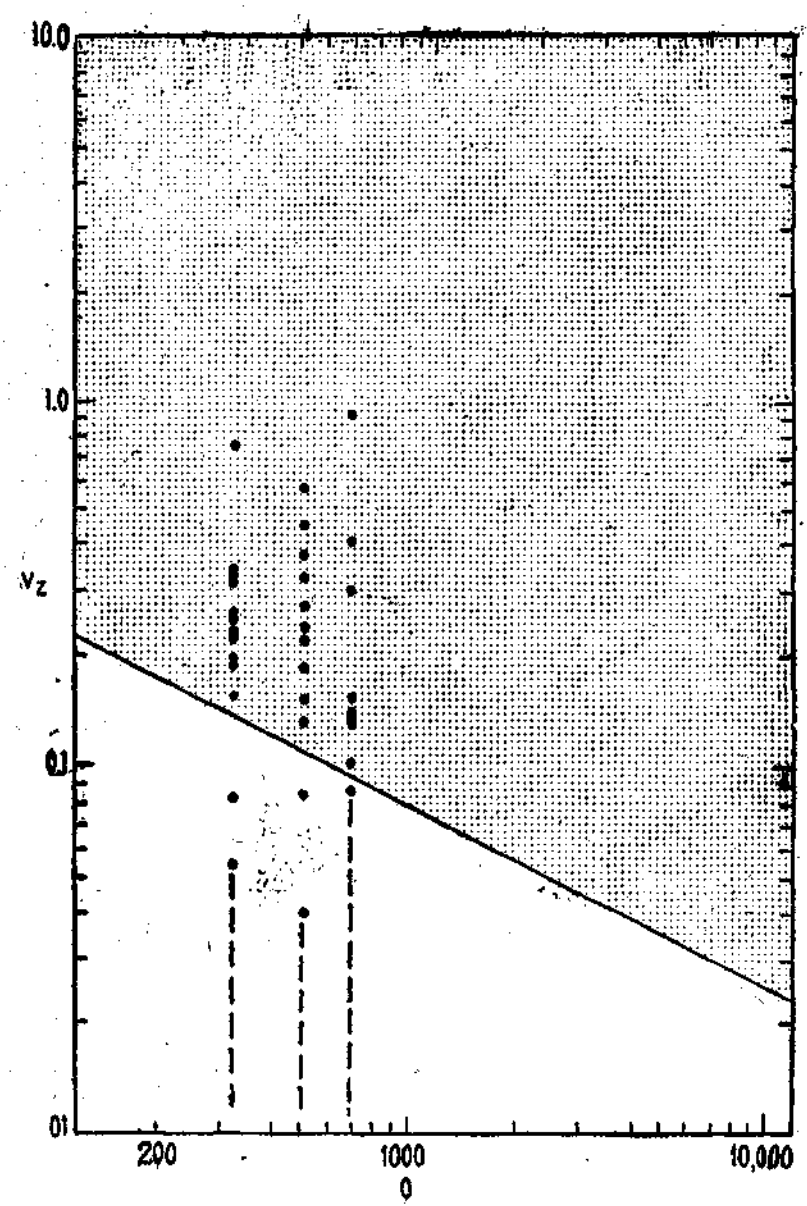

Fig. 6. Velocity gradient V_z against Schmidt number σ. The large dots represent the parameters of the observed sheets; the dotted lines indicate values of the parameters for which no sheets were observed. Here $\Omega = 0.98$ rad/s, $S = 0.00165 \pm 0.00017$ cm^{-1}, cylinder diameter 13.8 cm, disc diameter 10.1 cm.

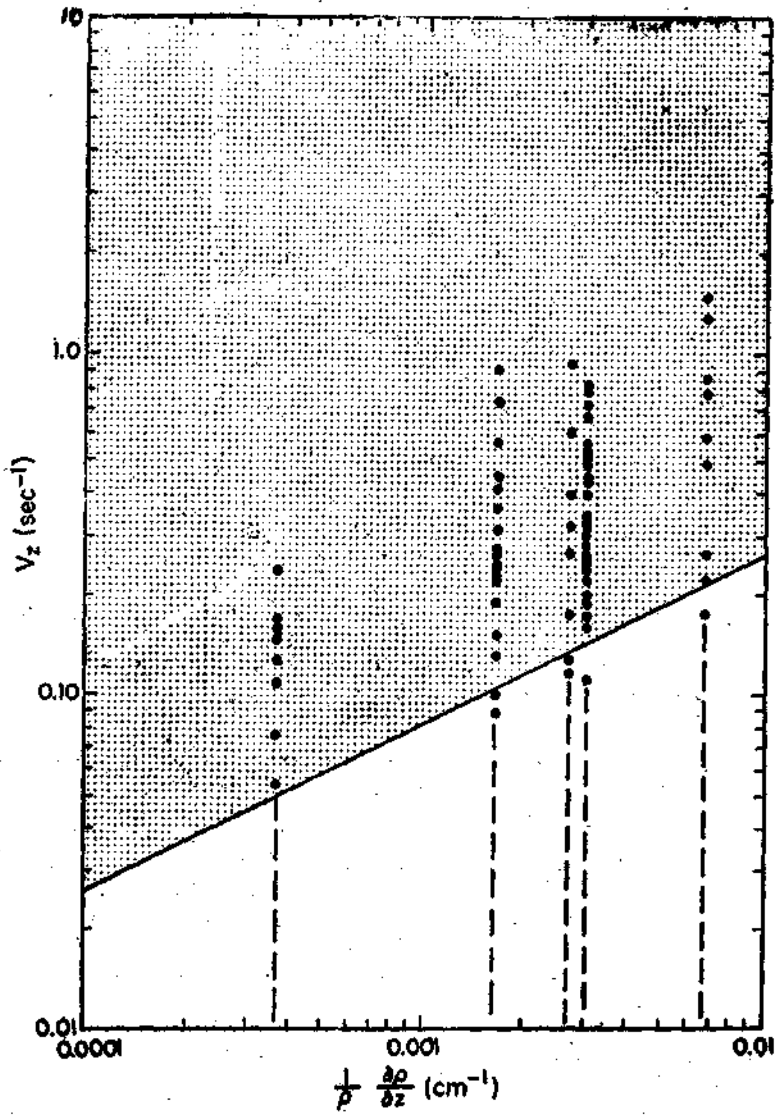

Fig. 7. Velocity gradient Vz against the stratification parameter S. The large dots represent the parameters of the observed sheets; the dotted lines indicate values of the parameters for which no sheets were observed. Here Ω = 0.99 rad/s, σ =700, T =20°C, cylinder diameter 13.8 cm, disc diameter 10.1 cm.

(a procedure similar to that used by Thorpe, *et al.*, 1969). We see that the theory predicts the magnitude of the distance reasonably well. The predicted dependence of the thickness on the velocity gradient apparently does not agree with the data available ; the unavoidable finite amplitude nature of the experiment may preclude such an identification.

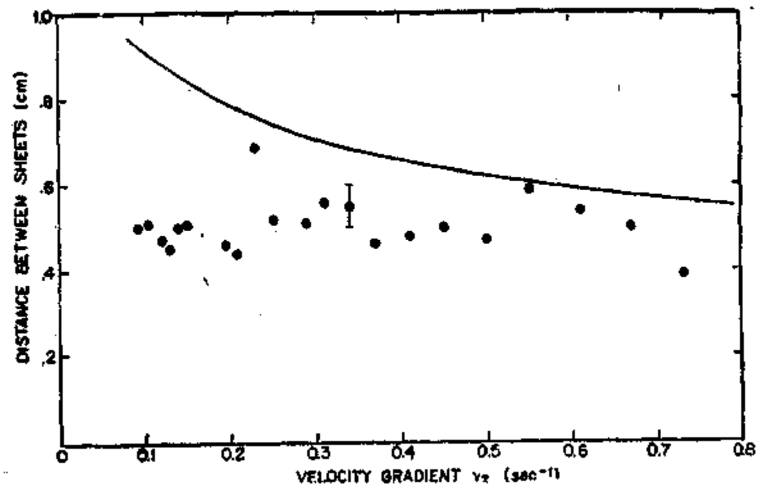

Fig. 8. Distance between the sheets as a function of the velocity gradient. The solid line represents one-half of the wave-length of the maximally growing disturbance of the theory. Here $\Omega = 0.926$ rad/s, $S = 0.00299 \pm 0.00030$ cm^{-4}, $\nu \approx 0.0095$ cm^2/s, $\sigma \approx 700$, T = 20°C, cylinder diameter 13.8 cm, disc diameter 10.1 cm.

A rough growth rate can be estimated by increasing the disc speed and measuring the time for the appearance of each new pattern. The decay rate is estimated by measuring the time for the disappearance of the pattern upon decrease of the disc speed. Fig. 9 illustrates the change of pattern in time as measured by the number of sheets visible for both an increase and a decrease of the disc speed. For these parameters, the theoretical growth rate yields a doubling time approximately one-tenth of the time observed for the full development of a given sheet. The decay time for a typical sheet (Fig. 9) is approximately 380 seconds, a time which is comparable to the diffusion time for salt across a distance equal to the sheet thickness (typicallyabout 1 mm).

Thus, the experimental results appear to be consistent with the viscous instability theory, although more accurate and extensive observations will be necessary for a complete confirmation. The parameters in the present system do admit the posibility of other instabilities, both symmetric and non-symmetric. The symmetric nature of the observed pattern preclude explanations on non-symmetric instability ; the length scales predicted from classic symmetric instabilities (Stone, 1966) are much larger than those observed in the experiment.

We have also observed sheets in an arrangement where the linear gradient lies below a layer of homogeneous density. In that case, the sheets appear at the top of the linear gradient, below the homogeneous layer. Moreover, sheets appear near the bottom of the container when the cylinder speed is changed markedly, then decay and disappear as the fluid reaches the new equilibrium rotation speed. The role of these density sheets in the non-linear stratified spin-up process is not yet understood.

The existence of horizontal sheets of density gradient in such a system suggests that the mechanism producing them may be responsible for some of the natural small-scale density structure discussed earlier in this paper. R.W. Stewart (Simpson and Woods, 1970) has suggested that some mechanism based on the difference between

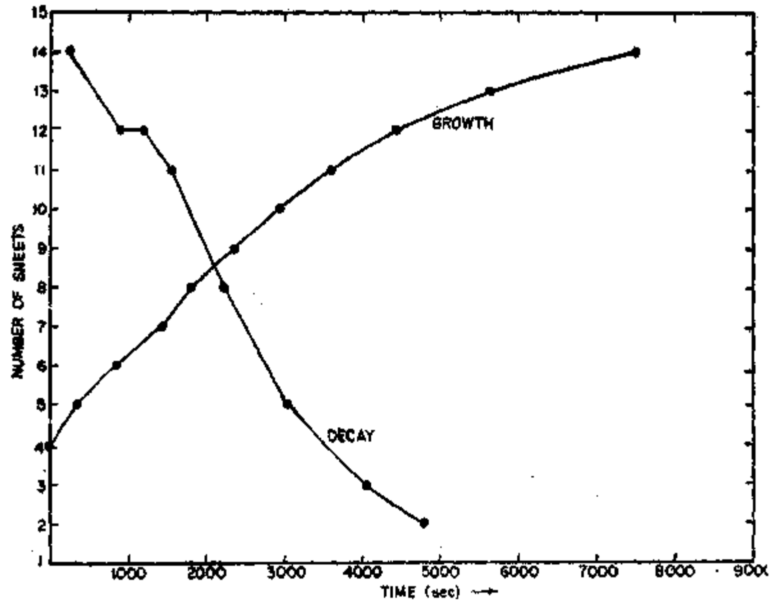

Fig. 9. Number of sheets against time. The growth curve was obtained by setting the Rossby number initially at -1 and allowing the pattern to develop. The decay curve was obtained by setting the Rossby number to zero after a full pattern had developed and then allowing the pattern, to decay. Here $\Omega = 0.957$ rad/s, $S = 0.0010 \pm 0.0001$ cm^{-1}, $\nu = 0.0095$ cm^2/s, $\sigma = 700$, $T = 20$°C cylinder diameter 13.8 cm, disc diameter 10.1 cm.

the turbulent transfer co-efficients of momentum and density is responsible for the density microstructure observed in fresh water lakes. The double diffusive destabilization mechanism, suggested here as an explanation for the laboratory results, may be relevant if the Prandtl number (or Schmidt number) is large enough in the natural systems. The instability would be operative on a scale determined by molecular diffusion should the molecular co-efficients and gradient Richardson number satisfy criterion (1). In addition, should the small scale turbulence in nature result in transfer of momentum and density in such a way that it could be characterised by turbulent transfer co-efficients, these larger co-efficients could be used in the criterion, and would increase the length scales.

To compare the results of observations in fresh water lakes and the ocean with the predictions of the theory, consider first the molecular diffusion processes. For heat, $\sigma = 7$, the instability requires $Ri < 1.5$, and a distance between sheets of $2\pi(\nu/\Omega)^{1/2} = 60$ cm is predicted. Although observations of the gradient Richardson number in regions of micro-structure are not presently available, both the records of Simpson and Woods and the spectral distribution of vertical wave number presented by Cox, *et al.* (Fig. 2) show considerable spectral intensity at this length scale. The typical temperature gradients of about 0.01 °C/cm observed by Simpson and Woods in regions of microstructure lead to instability, according to McIntyre's theory, if the velocity gradient is greater than 0.03 cm/s/cm, a value which is not unreasonable.

Turbulent diffusion co-efficients in the instability requirement yield $Ri < \nu_{turb}/4k_{turb}$; a criterion which is often met (see, for example, Taylor, 1931). However, since there are as yet no simultaneous measurements of microstructure and turbulent transfer co-efficients in lakes, the instability criterion in the turbulent case remains untested. The vertical scale of the layers can be estimated using typical magnitudes of the turbulent viscosity: 1 cm^2/s leads to a predicted thickness between the sheets of about 6m. This is the same order of magnitude as the thickness between sheets observed both in fresh water lakes and the ocean.

It is clear that the sharp gradients associated with microstructure can provide one way in which molecular diffusion can transport relatively large amounts of momentum, heat, or salt in the presence of a relatively small mean gradient. The existence and persistence of the sharp gradients in the density field in the laboratory experiment will allow a controlled test of this hypothesis.

The suggestion that layering may be produced by a large enough velocity gradient, even in the presence of large Richardson numbers, raises the possibility of experiments to test the hypothesis. Since the surface velocity gradients depend directly on the wind, measurement of microstructure before, during, and after times of large storms would be revealing. Moreover, seasonal measurements of microstructure in the Indian Ocean with its variable winds could form another useful test of the suggested theory.

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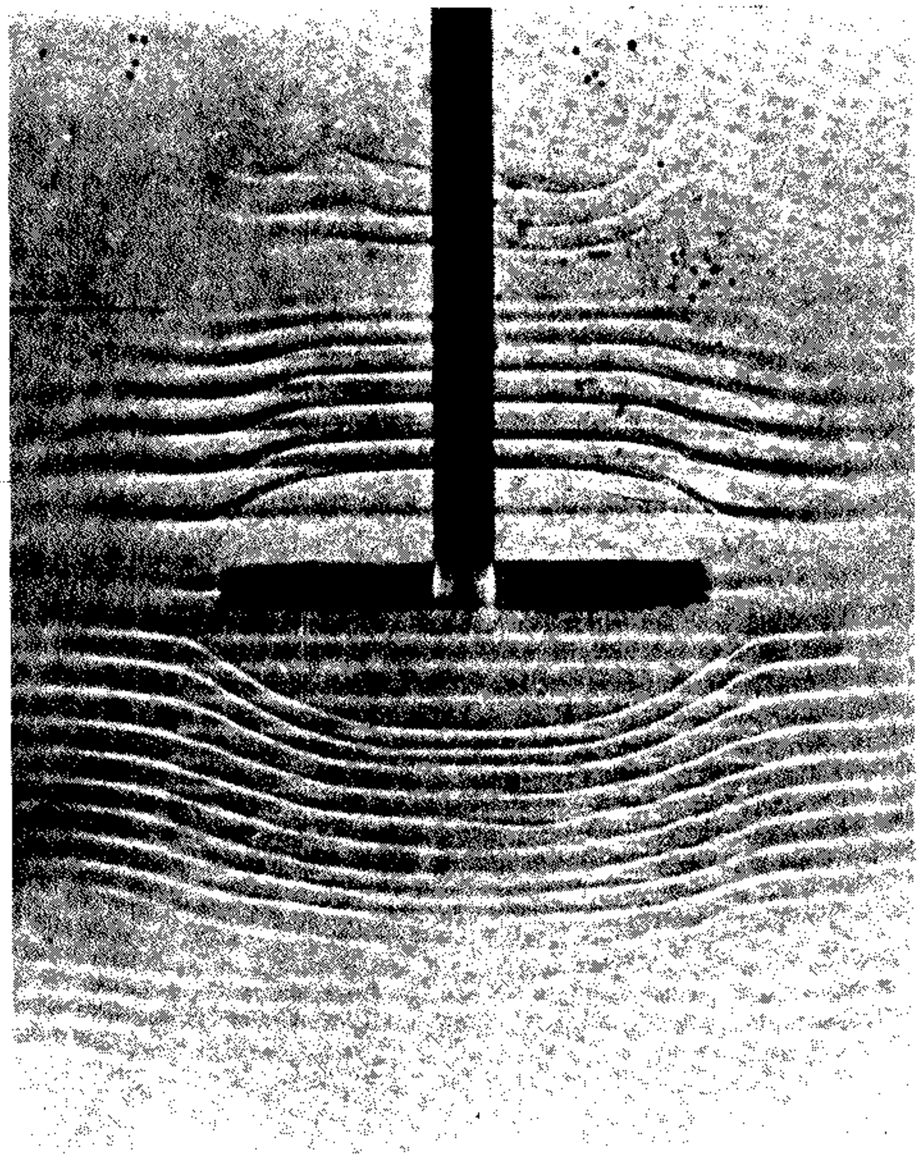

PLATE I. Shadowgraph of the density field in a rotating stratified fluid. The bright lines indicate regions of rapidly changing density. Here the Rossby number is 1, the cylinder angular velocity 1.05 rad/s, diameter of cylinder 27.8 cm, diameter of disc 10.1 cm, S = 0.0017 ± 0.0002 cm^{-1}, T = 20 C, σ = 720

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